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LETTER TO THE EDITOR

# Domain growth in the dilute random-field Ising model and the breakdown of self-similar dynamical scaling

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**Abstract.** Using Monte Carlo simulation, we have studied the kinetics of the domain growth in the dilute random-field Ising model with Glauber dynamics in two dimensions, following a quench from a very high temperature to a low-temperature unstable state. Our data strongly suggest a breakdown of self-similar dynamical scaling.

During the last decade considerable attention has been paid to various generalisations of the Ising model. For example, the random-field Ising model (RFIM) is defined by the Hamiltonian (see Imry (1984) and Villain (1985) for reviews)

$$\mathscr{H} = -J \sum_{\langle ij \rangle} S_i S_j + \sum_i H_i S_i \tag{1}$$

where the Ising spins $S_i$, $S_j$, etc, at the lattice sites $\boldsymbol{r}_i$, $\boldsymbol{r}_j$, etc, interact via the nearest-neighbour (NN) exchange interaction $J$. In addition, a random external magnetic field $H_i$ acts on each spin $S_i$. Both the discrete distribution

$$P(H_i) = 0.5\delta(H_i - H) + 0.5\delta(H_i + H) \tag{2}$$

and the continuous distribution

$$P(H_i) = \{1/[(2\pi)^{1/2}H]\} \exp(-H_i^2/2H^2)$$

have been considered in the literature.

Another interesting generalisation of the Ising model, the dilute Ising model (DIM), is defined by the Hamiltonian (see Stinchcombe (1983) for a review; see also Chowdhury and Stauffer (1986a, b))

$$\mathscr{H} = -\sum_{\langle ij \rangle} J_{ij} c_i c_j S_i S_j \tag{3}$$

where $c_i$, $c_j$, etc, are the probabilities that the sites $\boldsymbol{r}_i$, $\boldsymbol{r}_j$ are occupied by the Ising spins. We define

$$c_i = 1 \qquad \text{if the lattice site } \boldsymbol{r}_i \text{ is occupied} \tag{4a}$$

and

$$c_i = 0 \qquad \text{otherwise} \tag{4b}$$

so that $p$ is the average concentration of the spins in the system.

In this letter we shall investigate the domain growth law in the dilute random-field Ising model (DRFIM), whose Hamiltonian is

$$\mathscr{H} = -J \sum_{\langle ij \rangle} c_i c_j S_i S_j + \sum H_i S_i \tag{5}$$

where the symbols have the same meanings as stated above.

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We have studied the growth of ferromagnetic domains in the two-dimensional DRFIM following a rapid quench from a very high temperature to a temperature $T_f$. Although it is now generally believed that, in two dimensions, there is no transition from the paramagnetic to the ferromagnetic phase in the RFIM at any non-zero temperature, very large domains can grow at sufficiently low temperatures, as shown earlier by Gawlinski *et al* (1984, 1985). Some typical configurations at successively longer time scales following a quench are shown in figure 1. Since the system is non-self-averaging (Milchev *et al* 1986), we have averaged over a large number of configurations (quenches) $N_q$ in spite of the fact that our system sizes were quite large ($300^2$). The large size of the lattice was used to ensure the absence of any significant finite-size effect. The average size of the domains $R_m$ has been determined from the non-equilibrium magnetisation per spin (the details of the computational method will be reported elsewhere (Chowdhury *et al* 1986a)), i.e.

$$R_m^2 = pN\langle[(1/pN)\sum S_i]^2\rangle \tag{6}$$

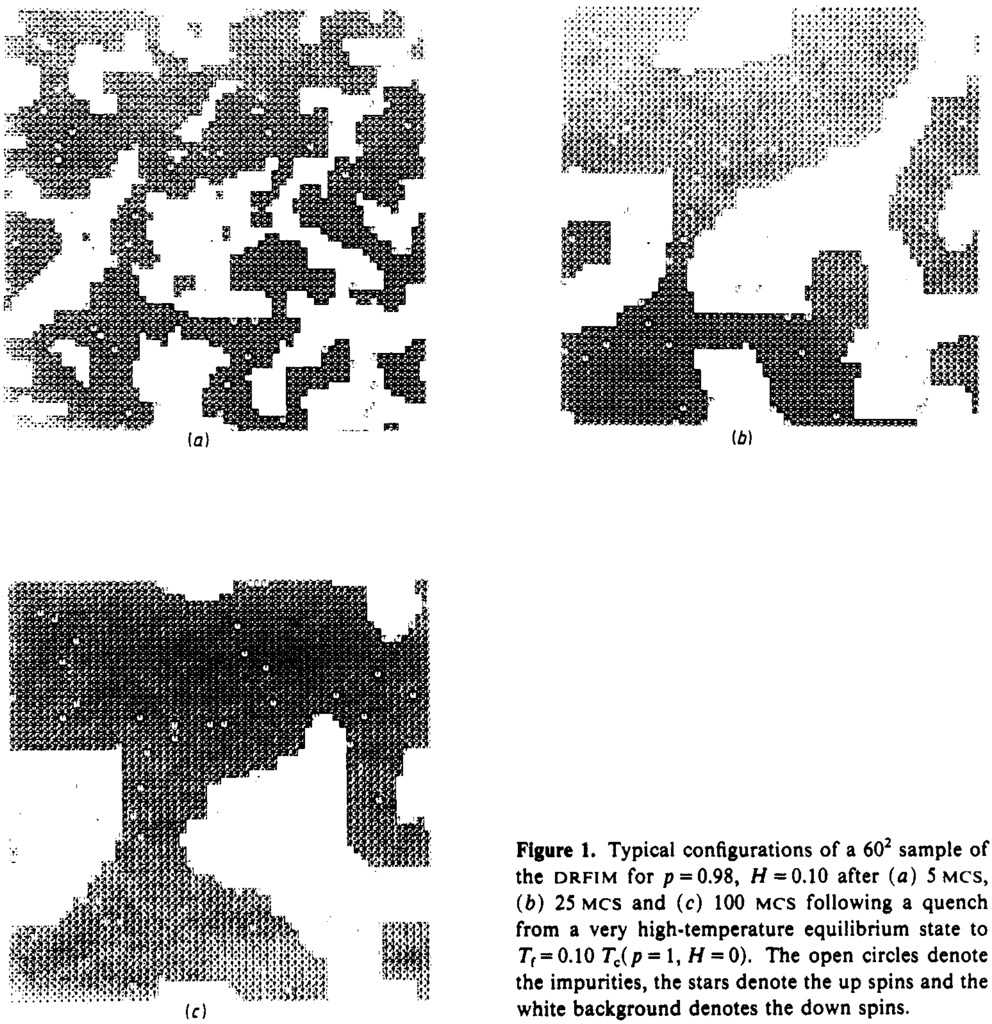

**Figure 1.** Typical configurations of a $60^2$ sample of the DRFIM for $p = 0.98$, $H = 0.10$ after (*a*) 5 MCS, (*b*) 25 MCS and (*c*) 100 MCS following a quench from a very high-temperature equilibrium state to $T_f = 0.10\ T_c(p = 1, H = 0)$. The open circles denote the impurities, the stars denote the up spins and the white background denotes the down spins.

as proposed by Sadiq and Binder (1984). The size of the domains is plotted as a function of time $t$ for various values of the random-field strength in figure 2. These curves are qualitatively similar to those in the case of the RFIM (Gawlinski *et al* 1984, Chowdhury and Stauffer 1985). The most crucial observation is that the growth of the domains is not only much slower than that in the pure Ising model in the absence of random field (Gawlinski *et al* 1984), but also slower than that observed in the dilute Ising (Chowdhury *et al* 1986b) and random-field Ising models (Gawlinski *et al* 1984, Chowdhury and Stauffer 1985). The so-called Allen-Cahn law (Allen and Cahn 1979, Kawasaki *et al* 1978), which describes the curvature-driven domain growth, states that $R_m^2 \propto t$. Clearly, this law breaks down in the DRFIM as should be expected. Also note that, although the growth in the DRFIM can be described by $R_m^2 \propto t^n$, the effective $n$ would decrease with the increasing strength of the random field.

In two dimensions the relation between the 'domain size' $R_m$ and the structure factor $S(k, t)$ is $R_m^2 = S(0, t)$, where $k$ is the wavenumber and $t$ denotes time. It is well known (see Gunton and Droz (1983) and Gunton *et al* (1983) for reviews) that for the pure Ising model in the absence of a random field, evolving from unstable states, the structure factor $S(k, t)$ follows the dynamical scaling form

$$S(k, t) = R^d f(kR(t)) \qquad (7)$$

in $d$ dimensions where $f(x)$ is the scaling function. The characteristic length scale $R(t)$ in (7) is, of course, time dependent. We have not explicitly studied the scaling function for the DRFIM. However, one can fit these data with a power law of the form $R(t) \sim t^n$ with the effective exponent $n$ varying with the parameters $H$ and $p$. Thus the usual self-similar dynamical scaling seems to be violated here. Although the

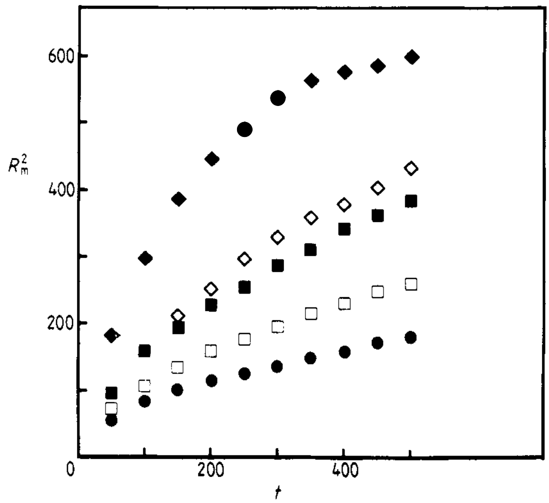


**Figure 2.** The characteristic size $R_m^2$ of the ferromagnetic domains in the two-dimensional DRFIM as a function of time (Monte Carlo step per spin). In this figure $p$ denotes the concentration of the Ising spins, $H$ is the strength of the random fields (defined through equation (2)) and $N_q$ is the number of quenches over which the corresponding data have been averaged. For $H = 0.05$ comparison of the data for $p = 0.995$, 0.99 and 0.975 shows $R_m^2$ to decrease drastically with decreasing $p$, at least for sufficiently long times. For $p = 0.995$, increasing $H$ reduces $R_m^2$. ●, $p = 1.0$, $H = 0.5$, $N_q = 50$; □, $p = 0.995$, $H = 0.5$, $N_q = 50$; ■, $p = 0.995$, $H = 0.4$, $N_Q = 75$: ◇, $p = 0.990$, $H = 0.3$, $N_q = 50$; ◆, $p = 0.975$, $H = 0.05$, $N_q = 50$.

equilibrium properties of the DRFIM are not known, there is little doubt that there is no long-range order at finite temperatures, as in the two-dimensional RFIM. As a consequence, we can envision the following scenario (Grant and Gunton 1984). Domains will grow until they reach some finite average size $\bar{R}_c$, which stops the process of ordering. Thus, there is a range of length scales, $R > \bar{R}_c$, which are inaccessible for scaling. Therefore, the system will no longer be dynamically self-similar. This has been shown explicitly for the RFIM. It would be of interest to confirm this for the DRFIM in a future Monte Carlo study.

In summary, both the dilution and the random field slow down the growth of the domains in the DRFIM. Our Monte Carlo data strongly suggest the breakdown of the self-similar dynamical scaling.

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